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Book Review: *Master of Modern Physics. The Scientific Contributions of H. A. Kramers*

Master of Modern Physics: The Scientific Contributions of H. A. Kramers. D. ter Haar, Princeton University Press, Princeton, New Jersey, 1998, pp. 288.

This book forms an important complement to Dresden's Kramers biography[1] although the setup and style is very different. The first half of the book takes the reader on a tour through Kramers' physics, discussing in some detail many of the papers contained in his Collected Scientific papers[2] and some more. The second part consists of twelve of his most important papers, translated into English where needed. Both make fascinating reading. Running through this are comments, many of these are inserted because ter Haar differs with Dresden on a number of essential points as mentioned in Chapter 1. Dresden used the "near misses" as leitmotif in his evaluation of Kramers work. Although this idea may have some merit, it left a distorted impression and it vastly underrated some of Kramers' major accomplishments. Nevertheless, after rereading parts of Dresden's biography, I think his book is a monument, since it brings out not only Kramers' contributions to physics and his craftsmanship, but also gives a lively description of Kramers as a human being.

Reading through all of Kramers work as presented by ter Haar is no minor task for two different reasons. First it is hard to be naive about quantum mechanics knowing how it all ended. Second the large variety of mathematical techniques displayed requires a lot of work for the reader. Finally I would like to mention the "made to order theories." The rich diversity of subjects Kramers tackled, puts him in the same class as Wigner.

Ter Haar's accomplishment to give a exegeses of many aspects of Kramers' work is the main contribution of this book. It is very readable.

The photo on the cover is an excellent choice. Kramers with his trademark bow tie and cigar (ready to throw some ashes in the surroundings) looking straight at you, although somewhat dreamy, if not a bit triste, but ready to pronounce another aphorism. It is a pity that librarians tend to

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throw away these cover pages; the picture does not appear in the body of the book.

Chapter 2 deals with the old quantum mechanics, i.e., pre-matrix or wave. Starting with Bohr's two equations ($E_2 - E_1 = hv$ and the action integral) it was extended by Sommerfeld for multiperiodical systems and by Bohr's correspondence principle. Next in line is Kramers' thesis which starts out with the Hamilton–Jacobi equations, then goes to the quantum mechanics of polarization and intensities of spectral lines, extensively described in the book. Very interesting is the first attempt to incorporate the electromagnetic radiation, which had failed [see Pais, Chapter 22]. This is now followed by several papers in which the electromagnetic radiation was fitted naturally in the old quantum mechanics, culminating in the Kramers Heisenberg papers (translated as reprint A).

Ter Haar stresses that Kramers fundamental contribution to the Bohr Quantum mechanics was that he showed, in several papers, how electromagnetism could be fitted into the old quantum mechanics.

One of the stories that keeps returning in the various biographies is the question why Kramers did not receive the Nobel prize. Although there has not always been a very clear relation between accomplishments and recognition, even if such entities could be quantified, the situation is further hindered by the fact that the coveted prize is given only once a year which in the time of the birth of quantum mechanics created somewhat of a backlog. The story goes somewhat as follows according to Korringa (see also Dresden, p. 292): after Kramers listened to a lecture by Heisenberg, he got the inspiration to write what became the K–H paper. He showed it to Heisenberg, and insisted that he be the coauthor. Mrs. Kramers who "always stood by her man" always said if you had not given in at that moment you would have had the Nobel prize. He shrugged it off; it was not part of his personality to get involved in a priority battle. He was not interested in a confrontation, and moreover he liked Heisenberg (Dresden, p. 458). The first unwritten rule for the Nobel prize is that the person has to be alive so his untimely death may have decided this question.

Chapter 3 deals with quantum mechanics as we know it now. This is mainly reflected in his book, which laid out the state of affairs before the war and contained many innovations, such as for instance the normalization of wave functions corresponding to continuous spectra and points overlooked by other quantum mechanics text books, such as the observation that the Schrödinger equation can formulated only if the forces acting on the particle can be derived from a potential energy, i.e., how fundamental the role of energy conservation is in quantum mechanics. His bad luck was that the book was in German and just before the war. It was reproduced during the war by the Alien property office (consequently he never received any royalties), and only translated in 1957 (thanks to ter Haar). It was reissued by Dover in 1964. Here and there were the "it is left to the reader" passages, some of which Kramers pretended he could not do himself! It was a struggle, particularly the second volume, but when things went well for the reader/student, it was an eye opener. I always liked the section on the symbolic method, starting innocently with the so called null-vector and building up to a very useful tool to calculate Clebsch Gordan coefficients and the like. The extremely original way in which he showed that the eigenvalue spectrum had to be discrete based on the properties of the Schrödinger equation and its boundary values was enlightening. It also lead in somewhat different form to the WKB method (with its clever use of the complex integral version of the Airy function to treat the behavior at the turning points) as well as the proof that periodic potentials lead to allowed and forbidden bands in the eigenvalue spectrum (Paper *G*). In this paper HAK derives a number of general properties of the trace of the transfer matrix (f) as function of the eigenvalues (E) of the periodic Schrödinger equation, i.e., $f(E)$ in a one dimensional system. This is a good example of the virtuoso mathematics he liked to perform. This is in sharp contrast to the "explanation of an experiment" type theory were the main effort lies in depicting a physical situation in terms of an equation.

The chapter on "special problems in quantum mechanics" starts with a number of aspects of sum rules, which seems to me a rather faded subject, but then come two gems: Kramers' degeneracy and charge conjugation, two subjects that are interlinked. There is a slight confusion in the notation, the paper which is translated as paper *E* uses *K*'s definition of the Pauli matrices, which was the custom in those years, while on p. 55 the present assignment of the components to the matrices is used. Also there is a spurious minus sign in the middle part of Eq. (14) (which would not matter too much if it were not that the whole proof hinges on plus and minus signs.). It is worth looking up Martin Klein's paper[3] who did the whole thing in a more stylized way, by extirpating the essential part out of it. In each case the core of the proof is to show that Hamiltonian remains the same after both complex conjugation is applied while simultaneously replacing the spin-operators by their negatives. The consequence being that systems with an odd number of electrons have a ground state that is at least doubly degenerate.

The last part of Section 3.5 uses similar ideas on the time-dependent Dirac equation, where Kramers shows its invariance under CC Charge Conjugation (reversing the sign of the electric charge) and the presence of holes.

Chapter 4. The electron in the electromagnetic field keeps haunting him. This lead to another Kramers first: the introduction of renormalization in quantum mechanics, i.e., the construction of a theory in which the experimental rather than the inertial mass of the electron appears. This idea was worked out in great detail by van Kampen in his thesis. Ter Haar gives quite some attention to this work and rightly so. With the renormalized hamiltonian Kramers was able to explain the Lamb shift.

Chapter 5 deal with Statistical Mechanics. As a pupil of Ehrenfest he had a special affection for statistical Mechanics. Dresden covered a part of the ground,[(4)] but ter Haar gives considerably more attention to this aspect.

It is less known that Kramers contributed significantly to statistical mechanics by introducing the thermodynamic limit, since this lies somewhat hidden in two papers. However he was well aware of its significance stating explicitly that the second solution containing the spontaneous magnetization was the result of this limit.

He wrote two papers on ferromagnetism, particularly the second is interesting. In this little known paper Kramers describes the high and low temperature behavior of a Heisenberg ferromagnet. He takes two steps, first the Boltzmann exponential is expanded and he argues that it suffices to consider the maximum term in this series. Then he evaluates expectation value of the powers of the Heisenberg spin hamiltonian. This expression is approximated for high and low temperatures. In the last case he obtains the spontaneous magnetization, using various combinatorial expressions.

Another milestone in statistical mechanics was the Kramers Wannier paper. Although the ultimate solution of the two dimensional Ising problem was obtained through a number of brilliant mathematical steps by Onsager, the foundation was laid in this paper containing two novel steps. The idea that adding another strip did not affect the previous state (i.e., the introduction of a transfer matrix) as well as the role of the eigenvalues of this matrix in the thermodynamic limit.

Finally there are two paper more or less on the side line, which still play a role in statistical mechanics: 3dr law and the didactical paper on the grand ensemble.

Particularly joyful reading is the first part of Chapter 6: the escape over a potential barrier, often referred to as the Kramers problem. (The original paper, entitled "Brownian motion in a field of force," is also reproduced in the second part of the book; paper *I*). In this section ter Haar gives an explanation of this problem with van Kampen cheering on the sidelines. While reading one sees Kramers busy showing how each section can be solved separately to bring out the essential part and then stringing at all together. Dresden mentions that after 50 years the paper was finally used for its intended purpose, the improved description of chemical reaction rates.

In this paper one finds the mathematical technique , which posthumously became very famous: the elimination of the slow variables. This came about in the barrier problem, which led to a Fokker Planck type equation which was solved by the observation that the large terms could be suppressed, under certain circumstances, by elimination of the momentum variable, which according to ter Haar has now become an industry under the title "adiabatic elimination of fast variables."

After the war the director of the laboratory was suspended to investigate whether he had collaborated and Kramers became the temporary director of the K.O. lab. In this period many experimental theses were finished under his guidance, which was more than a formality, since Kramers adorned many experimental results with explanatory theories. It shows there was a certain playfulness in his way of thinking. There is an amazing diversity in the explanation of experiments: the "made to order" theories so to speak.

The high precision measurements undertaken by Kistemaker required the determination of the pressure difference in a capillary containing a diffusing mixture of two gases: mercury vapor and helium. At one end of the capillary the gas was pure mercury vapor and at the other end it was pure helium. Kramers argued that the fluid velocity at the wall could not be zero. In the presence of this slipping velocity the correction is twice as large as without it. Experiments confirmed this. This is the only case were Kramers made a prediction for an experiment.

Experiments of Gerritsen lead Kramers to consider the stopping alpha particle in metals. The particle displaced electrons which form a cloud behind it, since there is a slight delay, this cloud drags the particle back. At the same time Kronig and Korringa came up with the idea of plasma waves (the first application of plasma in metals). The predictions were different and neither one in accordance with the experimental results. Another set of experiments by Gerritsen led to a theoretical explanation of the results of ionization experiments at low temperatures, since his results differed from the Jaffé-theory. Another issue that came up through the laboratory were the explanation of the vibrations of a gas column in low temperature experiments. Although not quite successful it laid the groundwork for this effect.

Going through and reporting on its contents is like "surfing" through major parts of theoretical physics in a delightful manner. I hope the reader of this review feels inclined to do the same.

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